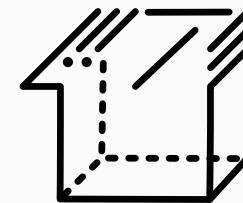


# Towards Computational UIP in Cubical Agda

or: making proof assistants nicer to use by **filling squares!**



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Yee-Jian Tan (j.w.w. Andreas Nuyts, Dominique Devriese)

DRADS 2026

16 March 2026

# Why Proof Assistants Matter?

Proof assistants (e.g. Agda , Rocq , Lean ) verify mathematical proofs.

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$\forall i : I, \text{isNice} (P i)$  holds.

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 **RustCompCert** [Wu+26]

(ongoing, 2026) — end-to-end verified Rust compiler, including a verified borrow checker.

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Assumes the **univalence axiom**: equivalent (isomorphic) types are equal. (If they behave exactly the same way, they must be equal). Buy 1 free 2!

Implemented in Agda as **Cubical Agda**.



Univalence derived from **Glue** types.

# ~~Homotopy Type Theory~~

Let's do **paths** and **squares** and **cubes**!  <sup>1</sup>

---

<sup>1</sup>[Coh+17]

# Equality as paths: geometric interpretation

**Type** : a space

**Terms** : points in that space

**Equality proof** : a path between points

An example in natural numbers  $\mathbb{N}$ :

$\mathbb{N}$  is a space: 0, 1, 2, ... are its points.



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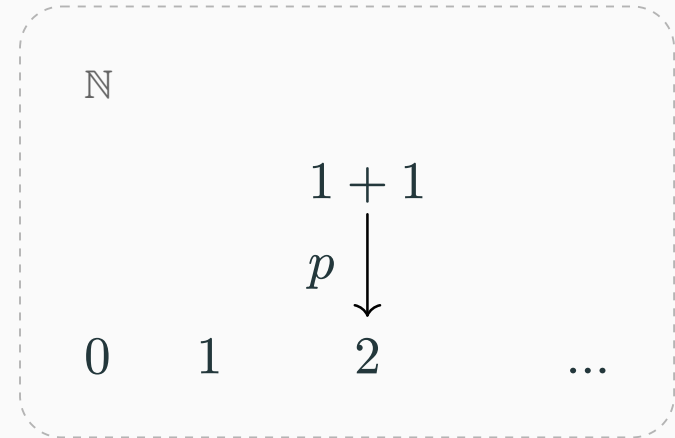
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An example in natural numbers  $\mathbb{N}$ :

A proof that  $1 + 1 = 2$  is a path  $1 + 1 \xrightarrow{p} 2$  in  $\mathbb{N}$ .<sup>1</sup>

Reversing  $p$  gives you  $2 = 1 + 1$  (symmetry).



<sup>1</sup> $\mathbb{N}$  is actually a **set** (h-set): all paths are refl, and  $1 + 1, 2, 2 + 0$  are the same point. Shown as distinct here for illustration only.

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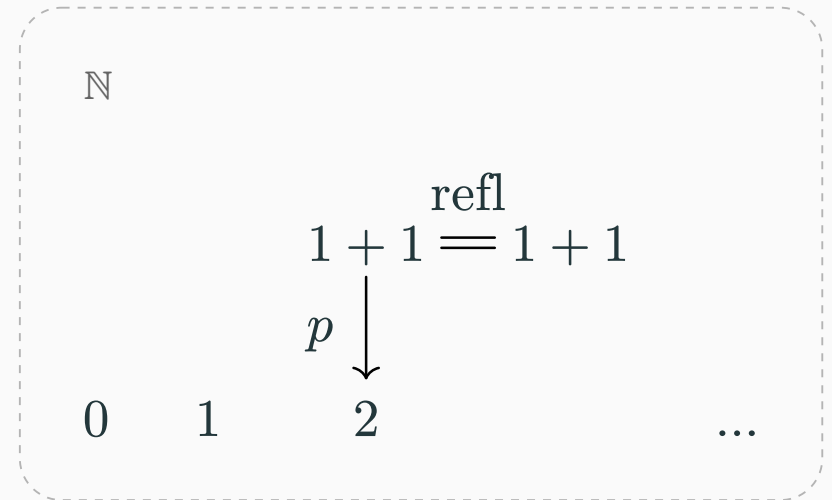
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An example in natural numbers  $\mathbb{N}$ :

What about  $1 + 1 = 1 + 1$ ? That is the constant path, **refl** (reflexivity).



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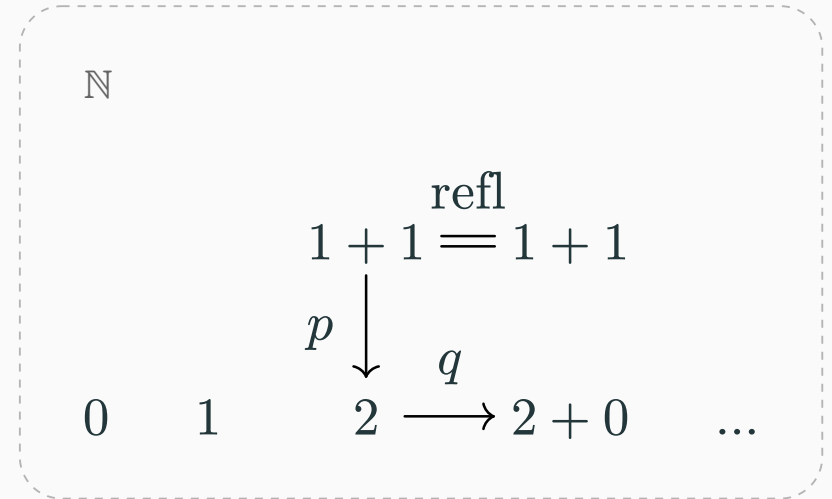
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An example in natural numbers  $\mathbb{N}$ :

In Cubical Agda, you can compose 3<sup>1</sup> paths:

- $p : 1 + 1 = 2$  and
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- $\text{refl} : 1 + 1 = 1 + 1$

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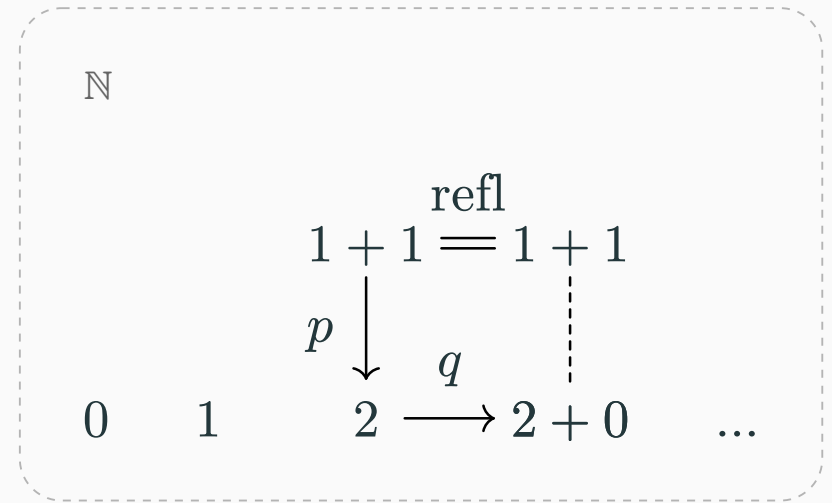
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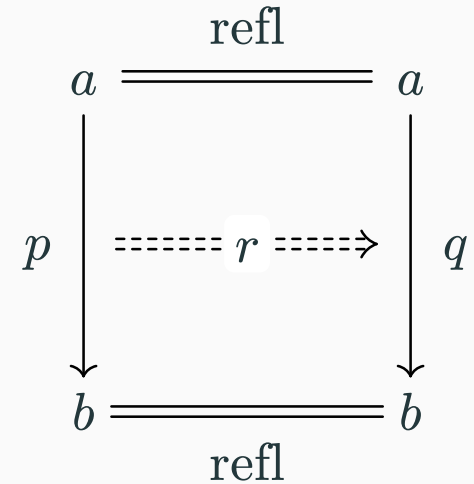
**Equality proof** : a path between points

## Observation

For an arbitrary type  $A$ , two paths  $p, q : a =_A b$  can themselves be **equal**: witnessed by a **square**.

Squares can be equal too: then it becomes... a **cube**!

And **cubes** can be equal too... (you get the idea)



# UIP and Square-Filling

In general, two paths  $p, q : a = b$  need not be equal, but many structures (e.g.  $\mathbb{N}$ ,  $\mathbb{Z}$ ) are like sets: paths are always unique. (Easier!)

## Uniqueness of Identity Proofs (UIP) condition

“There is at most one proof of  $x =_A y$  for any  $x y : A$ .”

Incompatible with one key ingredient of HoTT: **univalence**!

Fortunately compatible with Cubical Agda without *Glue* types: the source of **univalence** in Cubical Agda. We implemented a `--cubical=no-glue` variant of Agda [Tan25].

**Geometrically:**

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**UIP**: any square with two **reflexive** sides has a filling.

$$\begin{array}{ccc} x & \xlongequal{\text{refl}} & x \\ p \downarrow & \text{UIP} \Rightarrow & \downarrow q \\ y & \xlongequal{\text{refl}} & y \end{array}$$

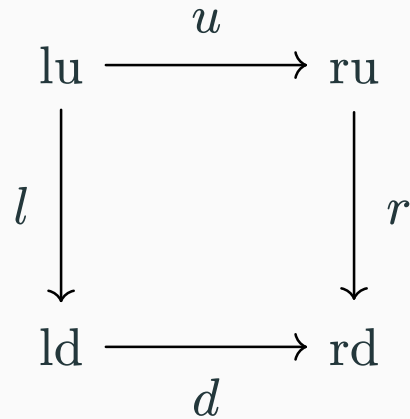
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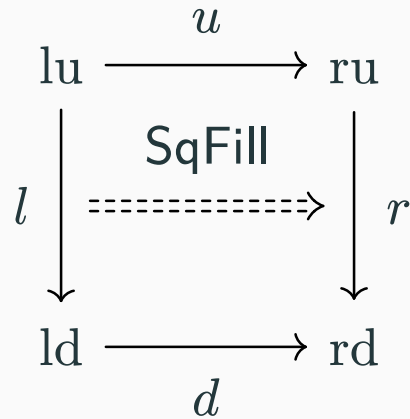
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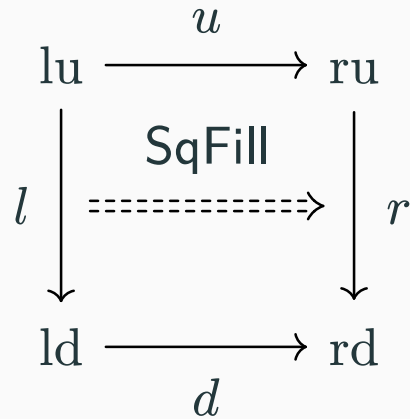
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Surprisingly:  $\text{UIP} \leftrightarrow \text{SqFill}$  are **equivalent**, but SqFill is easier to work with!

Genius idea  : just add in **SqFill** everywhere!

## Bad news: SqFill doesn't compute (naively)

Naïvely adding SqFill as a **postulate** blocks computation. If SqFill appears in a term:

... **SqFill** ( $\mathbb{N} \rightarrow (\mathbb{N} \times \text{Bool})$ ) ...

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Everything else around it evaluates, but it doesn't.

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**Proof by induction on the type:** reduces to base cases, and computes away.

# Live Demo: Cubical-UIP in Agda

**Cubical Agda:** SqFill as a stuck axiom:

```
-- postulated, never reduces
postulate
  prim^sqFill : ∀ (A : Type) → SqFill A

-- doesn't hold automatically!
test-computes : sqFill-block refl refl ≡ refl
test-computes = {! refl !} -- LHS does not reduce
```

**Cubical-UIP:** SqFill that computes:

```
-- dispatches by type, always reduces
primitive
  prim^sqFill : ∀ (A : Type) → SqFill A

-- computes!
test-computes : sqFill-block refl refl ≡ refl
test-computes = refl
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Live Demo with **Cubical-UIP!**

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Base cases: types where equality is obviously unique, a proof is directly given.

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**Proof by induction on the type:** the type checker assembles the proof automatically.

# Summary: SqFill for each type former

Full [Agda proofs](#) with beautiful [diagrams](#) [TND25] available.

Type former	Difficulty
$\Pi(a : A).B a$ (functions) <sup>1</sup>	: Trivial: pointwise, no path manipulations needed <sup>2</sup>
$\Sigma(a : A).B a$ (dependent pairs) <sup>3</sup>	: <b>Involved</b> : complex path manipulations <sup>4</sup>
$A \uplus B$ (coproducts)	: Standard: encode, manipulate, decode
$\mathbb{N}$ (natural numbers)	: Standard: encode, manipulate, decode
$a =_A b$ (path types)	: Simple: a single path composition <sup>5</sup>
$\mathcal{U}$ (the universe)	: <b>Open question</b> : see next slide

---

<sup>1</sup>Non-dependent ( $A \rightarrow B$ ) is a special case: also trivial.

<sup>2</sup>No Kan operations (hcomp/transport) required.

<sup>3</sup>Non-dependent ( $A \times B$ ) is a special case that's trivial.

<sup>4</sup>Requires transport-fill-align: transport along a line of types, fill a box, then align the result via hcomp.

<sup>5</sup>A single hcomp suffices.

## Open question: what about the universe? 🪐

So far: SqFill computes for **concrete** type formers:  $\Pi$ ,  $\Sigma$ ,  $A \uplus B$ ,  $\mathbb{N}$ ,  $a =_A b$ .

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Two approaches, both open:

- **Approach A**: treat  $\mathcal{U}$  as an inductive type with one constructor per type former — inspect it the same way we inspect  $A \times B$ ,  $A \rightarrow B$ , etc.<sup>1</sup>
- **Approach B** (speculative — merely an idea at this stage): **SqFill** becomes a new type former itself, with the so-called **Weld** types<sup>2</sup>.

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<sup>1</sup>Formally: an inductive-recursive (IR) universe with a typecase operator, in the style of Dybjer [Dyb00] / Dybjer-Setzer [DS99] / XTT [SAG22]. This makes the type theory anti-classical [CD18, Hur10], invalidating excluded middle — but univalence is already disabled.

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*Ask us after the talk if you want to know more!*

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TL;DR: Want all of the following?

- **Funext**: pointwise equal functions are equal
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- ✓ `--cubical=uip`: SqFill computation rules for **Pi**, **Sigma**, **Nat**, **Bool**, **Unit**

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**Work in progress:** 🚧

- Universe case: IR universe via typecases
- Path types and generic inductive types
- Canonicity and Normalisation
- Evaluate advantage in projects that needed Computational UIP [CNT26, HC21]

**Thank you!**

Questions?

<https://github.com/yeejian-tan/agda/tree/cubical-uir>

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